Determination of the six DOF parameters of CAD-based objects

Dr. X Wang & Dr. T A Clarke
Dept. of E.E. & I.E., City University
Northampton Square, London. EC1V 0HB
United Kingdom
Email: x.wang@city.ac.uk

Key Words: six DOF parameters, co-ordinate transformation, CAD-based object

ABSTRACT

Photogrammetric measurement systems can be used to simultaneously obtain 3-D co-ordinates of targeted points. However, obtaining 3-D co-ordinates of spatial points is not usually the final objective. The measured 3-D data may be used for many other purposes, such as determining the position and orientation (six degrees of freedom) of an object with respect to a specified co-ordinate system or the relative position and orientation of two or more objects. In either case CAD descriptions and tolerance information are generally available. This paper will discuss how the 3-D data can be matched with object CAD descriptions to determine object locations and how the errors are propagated from measurement source to the estimated results. The optimum design of a measurement system to meet a defined tolerance budget will also be discussed.

1. Introduction

As future manufacturing becomes more automated a higher reliance on measurement systems as a process enabling technology is becoming clear. In the aerospace area large scale metrology systems are often required. One of the few techniques available to measure the relative orientation of multiple components is photogrammetry.

A generic requirement in manufacturing is the placement of components into desired locations prior to fixing. If photogrammetry is to be used it is necessary to relate CAD information for each component to the location of targets on the object. It is then possible to determine the location of the components with respect to any desired location. While targets are important, they are merely means to an end. This paper discusses a method of computing the location of the objects without the usual intermediate step of computing the 3-D points and then relating these to the CAD information.

To illustrate this process, consider the assembly process illustrated in figure 1. A component (a leading edge rib) is observed by two or more cameras together with a spar. Targets in known positions with respect to the spar and the rib allow provide enough information to compute the relative orientation of the two components. This information, together with a knowledge of the relationship between the measurement system and the robot enable the robot to be controlled to move the rib into its desired location ready for drilling or fixing operations.

Figure 1. A possible advanced manufacturing process – the assembly of the leading edge of a airplane wing

2. Definition of terms

An object that is moved from one location to another can be described by six degrees of freedom parameters (3 translations x_t, y_t, z_t and 3 rotations α, β, γ) with respect to a given co-ordinate system XYZ as shown in figure 2. The task is to determine the six transformation parameters.

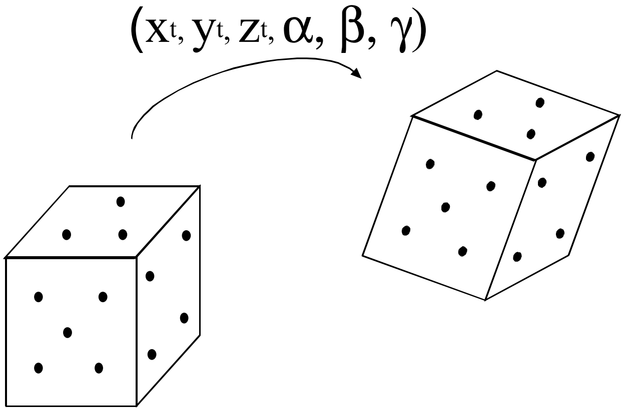


Figure 2. A simple object transformation example

In a three-dimensional homogeneous co-ordinate representation, the translation of a point from position $P_1 = [X_1 \;\; Y_1 \;\; Z_1]^t$ to position $P_2 = [X_2 \;\; Y_2 \;\; Z_2]^t$ can be expressed as $P_2 = P_1 + T$ where $T = [x_t \;\; y_t \;\; z_t]^t$ is a vector of translation distances for the co-ordinate direction X, Y and Z. A three-dimensional rotation of a point can be specified around a line in space. The most convenient rotation axes to deal with are the three co-ordinate axes. Conventionally positive rotation directions about the co-ordinate axes are counter-clockwise (Hearn & Baker, 1994), when looking towards the origin from a positive co-ordinate position on each axis. The 3-D rotation transformation can be expressed as

$$P_2 = R \cdot P_1$$

in which R is 3 by 3 orthogonal rotation matrix related to the three rotational angles α, β and γ. The six degrees of freedom (DOF) are defined as a translation followed by a rotation, therefore the transformation equation can be written as

$$P_2 = R \cdot P_1 + T$$

The six DOF transformation is illustrated in figure 3.

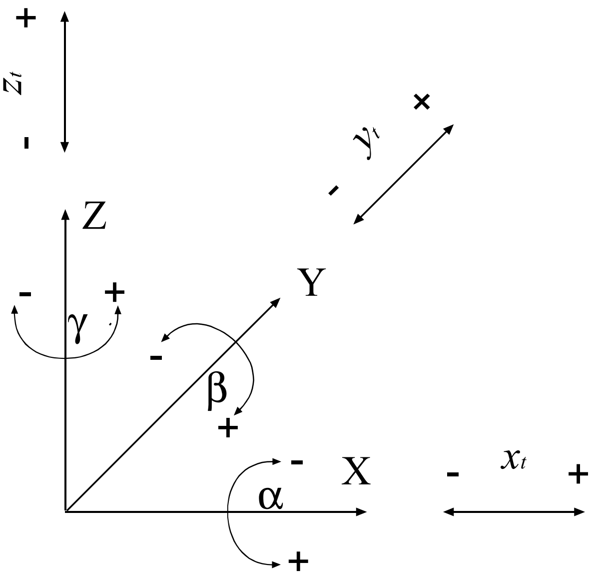


Figure 3. The definition of the six degrees of freedom parameters

With a photogrammetric measurement system the location of the targets on the object can be determined in a cartesian co-ordinate system. These targets could be special features such as centre of holes, corner, or more typically retro-reflective targets specifically applied for photogrammetric purposes.

2. The two step method

The normal procedure of estimating the six DOF parameters would be (1) determine the 3-D co-ordinates of the targets on the object before and after transformation, (2) compute the six transformation parameters using on the two sets of co-ordinates.

The 3-D co-ordinates of the target on the object can be obtained by a photogrammetric bundle adjustment or direct intersection. The 2 sets of co-ordinates before and after the movement must relate to the same co-ordinate system.

Suppose that *n* corresponding targets on the object have be located in 3-D with respect to the co-ordinate system XYZ, and they will be used to determined the six transformation parameters. The co-ordinate transformation functional model for a single point can be expressed by equation 1 as

$$\boldsymbol{F}_i = \boldsymbol{R} \cdot \boldsymbol{P}_{1i} + \boldsymbol{T} - \boldsymbol{P}_{2i} = 0 \qquad (1)$$

(i = 1, 2, …, n)

where

$\boldsymbol{P}_{1i} = [\boldsymbol{X}_{1i}\ \boldsymbol{Y}_{1i}\ \boldsymbol{Z}_{1i}]^t$ and $\boldsymbol{P}_{2i} = [\boldsymbol{X}_{2i}\ \boldsymbol{Y}_{2i}\ \boldsymbol{Z}_{2i}]^t$

denote the 3-D co-ordinates of target *i* on the object before and after transformation, respectively. This is a six parameter rigid transformation without scale change. Each corresponding target will give rise to three equations. With a minimum number of three targets the six transformation parameters can determined by a least squares estimation. Linearizing the functional model (2) and considering, for the sake of generality, both $\boldsymbol{P}_{1i}$ and $\boldsymbol{P}_{2i}$ as observations yeilds:

$$\boldsymbol{A}_i \Delta \boldsymbol{u} + \boldsymbol{B}_i \boldsymbol{l}_i = \boldsymbol{C}_i$$

where $\boldsymbol{u} = [x_t\ y_t\ z_t\ \alpha\ \beta\ \gamma]^t$ is a column vector of six transformation parameters and $\boldsymbol{l}_i = [\boldsymbol{X}_{1i}\ \boldsymbol{Y}_{1i}\ \boldsymbol{Z}_{1i}\ \boldsymbol{X}_{2i}\ \boldsymbol{Y}_{2i}\ \boldsymbol{Z}_{2i}]^t$ is a column vector of 2 sets of co-ordinates before and after transformation and $\boldsymbol{C}_i = -\boldsymbol{F}_i$. $\boldsymbol{A_i}$ and $\boldsymbol{B_i}$ are Jacobi matrices, i.e.,

$$\underset{3\times 6}{\boldsymbol{A}_i} = \frac{\partial f_i}{\partial \boldsymbol{u}} = \left[\underset{3\times 3}{\boldsymbol{I}}\ \ \underset{3\times 1}{\boldsymbol{R}_\alpha \boldsymbol{P}_{1i}}\ \ \underset{3\times 1}{\boldsymbol{R}_\beta \boldsymbol{P}_{1i}}\ \ \underset{3\times 1}{\boldsymbol{R}_\gamma \boldsymbol{P}_{1i}} \right],$$

$$\underset{3\times 6}{\boldsymbol{B}_i} = \frac{\partial f_i}{\partial \boldsymbol{v}} = \left[\underset{3\times 3}{\boldsymbol{R}}\ \ \underset{3\times 3}{-\boldsymbol{I}} \right]$$

in which $\boldsymbol{R}_\alpha, \boldsymbol{R}_\beta, \boldsymbol{R}_\gamma$ are the partial derivatives of R with respect to the three rotation angles α, β, γ respectively. With *n* corresponding targets on the object before and after transformation, the linearized functional model becomes

$$\boldsymbol{A} \Delta \boldsymbol{u} + \boldsymbol{B} \boldsymbol{v} = \boldsymbol{C}$$

where

$\boldsymbol{A} = [\boldsymbol{A}_1\ \boldsymbol{A}_2\ \cdots\ \boldsymbol{A}_n]^t$, $\boldsymbol{B} = \boldsymbol{diag}[\boldsymbol{B}_1\ \boldsymbol{B}_2\ \cdots\ \boldsymbol{B}_n]$, $\boldsymbol{C} = [\boldsymbol{C}_1\ \boldsymbol{C}_2\ \cdots\ \boldsymbol{C}_n]^t$ and $\boldsymbol{l} = [\boldsymbol{l}_1\ \boldsymbol{l}_2\ \cdots\ \boldsymbol{l}_n]$. According to Mikhail & Gracie (1981) the six transformation parameters are estimated by

$$\Delta \boldsymbol{u} = (\boldsymbol{A}'(\boldsymbol{B}\boldsymbol{Q}_l\boldsymbol{B}')^{-1}\boldsymbol{A})^{-1}\boldsymbol{A}'(\boldsymbol{B}\boldsymbol{Q}_l\boldsymbol{B}')^{-1}\boldsymbol{C},$$

and the cofactor matrix of the estimated parameters is

$$\boldsymbol{Q}_u = (\boldsymbol{A}'(\boldsymbol{B}\boldsymbol{Q}_l\boldsymbol{B}')^{-1}\boldsymbol{A})^{-1}$$

where $\boldsymbol{Q}_l$ is a cofactor matrix of the 6n observed co-ordinates (3n co-ordinates before the transformation and 3n after). This is a general case of least squares estimation which requires sophisticated computations that may not be suitable for real-time applications. There are several different levels of simplifications. For instance, the two sets of co-ordinates can be treated as independent observations, i.e., there is no correlation between them, therefore the cofactor matrix $\boldsymbol{Q}_l$ becomes a block diagonal matrix. It can also be assumed that there is no correlation between co-ordinates in each set and they are all equally weighted, so the cofactor matrix $\boldsymbol{Q}_l$ will simply be a scalar. It can even be assumed that $\boldsymbol{P}_1$ is a vector of constants, while $\boldsymbol{P}_2$ is a vector of observations. In this case equation (1) becomes a set of typical observation equations therefore can be solved easily. The simplest case would be to assume that both $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are constants. So the least squares process can be simplified considerably. The estimated values of the six transformation parameters may not be influenced very much due to these assumptions if it is a strong convergent network with a large number of redundancies (Cooper, 1999). But the cofactor matrix of the estimates will surely be effected significantly. In many industrial applications it is important to estimate the values accurately and it is equally important to know the statistics of the estimates for quality controls. Therefore the full solution of the six DOF is required in terms of the value and the statistics.

3. An alternative method

Instead of solving 3-D co-ordinates and the six transformation parameters in two steps, an alternative method is to combine the photogrammetric measurements with transformation constraints and solve for the six transformation parameters directly. To achieve this objective the relationship between image observations and the transformation parameters need to be established. The functional model which links the 3-D co-ordinate and the image observations is based on the collinearity condition and can be written as

$$\boldsymbol{f}(\boldsymbol{P}, \boldsymbol{S}) = \boldsymbol{l}$$

where P denotes the 3-D co-ordinates, S denotes the camera parameters and l denotes the image observations. In real-time on-line photogrammetric measurement applications the camera interior parameters are normally calibrated and the image observations are corrected accordingly. So S denotes only the camera exterior parameters. The equations for solving the 3-D co-ordinates before and after the transformation, considering the relationship between P_1 and P_2 given by equation ???, can be written as

$$\boldsymbol{f}_1(\boldsymbol{P}_1, \boldsymbol{S}_1) = \boldsymbol{l}_1 \qquad \text{(a)}$$
$$\boldsymbol{f}_2(\boldsymbol{R}, \boldsymbol{T}, \boldsymbol{P}_1, \boldsymbol{S}_2) = \boldsymbol{l}_2 \qquad \text{(b) Why a and b?}$$

In the above equations the transformation parameters R and T can be solved directly. Since both equations take the typical form of observation equations, the transformation parameters can be computed easily by least squares and the cofactor matrix of the estimated parameters is directly linked with image observations l_1 and l_2. It is important to recall that the 3-D co-ordinates at the two positions (before and after the transformation) must be respected to the same co-ordinate system and the estimated transformation parameters will be related to this co-ordinate system.

A typical way of defining a co-ordinate system is to use control points. Suppose that some control points have been measured accurately and the co-ordinate system XYZ is defined by these control points. These control points can then be used as constraints in the least squares estimation process (Cooper, 1987). In this way the 3-D co-ordinates will be tied to the same co-ordinate system XYZ.

For an on-line application it is a common practice to determine camera exterior parameters by a bundle adjustment using a reference target array and then compute the 3-D co-ordinates by iterative intersection based on the collinearity equations. Under this condition the camera exterior parameters in equation (a) and (b) will be constants. The unknown parameters left to be solved in equation (a) and (b) are P_1 (the 3-D co-ordinates before the transformation) and (R, T) (the six transformation parameters).

In an industrial environment the components for assembly are normally CAD-based object, i.e., the 3-D data are ready to be used for the assembly tasks. Therefore retro-reflective targets may be attached to the object at the known locations or be measured. For instance targets can be put on adapters fitting into holes or corners on the CAD-based object. Figure 4 illustrates the CAD information relating to a leading edge component.

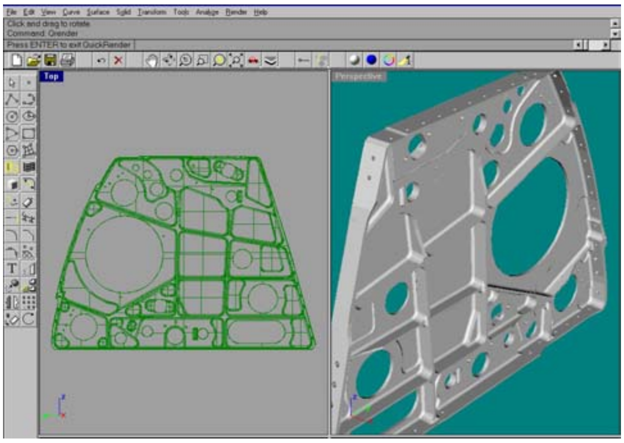


Figure 4. CAD information relating to a leading edge wing component

The 3-D co-ordinate of these targets in the initial position can be expressed by

$$\boldsymbol{P}_0 = [\boldsymbol{X}_0 \ \ \boldsymbol{Y}_0 \ \ \boldsymbol{Z}_0]^t$$

From the initial position to position k the six transformation parameters can be determined from equation (b), i.e.,

$$\boldsymbol{f}_k(\boldsymbol{H}, \boldsymbol{R}, \boldsymbol{P}_0, \boldsymbol{S}_k) = \boldsymbol{l}_k$$

The functional models take the form

$$\begin{cases} \boldsymbol{x}_k = -\boldsymbol{c}_f \dfrac{\boldsymbol{M}_1(\boldsymbol{R}\boldsymbol{P}_0 + \boldsymbol{T} - \boldsymbol{H})}{\boldsymbol{M}_3(\boldsymbol{R}\boldsymbol{P}_0 + \boldsymbol{T} - \boldsymbol{H})} \\ \boldsymbol{y}_k = -\boldsymbol{c}_f \dfrac{\boldsymbol{M}_2(\boldsymbol{R}\boldsymbol{P}_0 + \boldsymbol{T} - \boldsymbol{H})}{\boldsymbol{M}_3(\boldsymbol{R}\boldsymbol{P}_0 + \boldsymbol{T} - \boldsymbol{H})} \end{cases}$$

In above equations (x_k, y_k) are image observations of the targets on the object in position k. M_1, M_2 and M_3 are the first, second and third rows of the rotation matrix M witch is determined by the three rotational angles (ω, ϕ, κ) of the cameras. H is a column vector with three elements (X_L, Y_L, Z_L) witch defines the position of the camera perspective centres and c_f is the focal length (or more accurately the principal distance) of the cameras. The six transformation parameters can be solved directly from equation ?. Since it is a typical observation equation the least squares estimation will take its simplest form and the covariance matrix of the estimates can be obtained easily. Therefore it is convenient to assess the capabilities of the measurement network under various conditions by simulation and practical tests. Some simulation test results are given later in this paper.

4. An example of single camera solution

It is possible to determine the six DOF of an object using a single camera provided that the 3-D data of the targets on the object are given. This is similar as space resection where the camera exterior parameters can be solved using spatial control points. If it is assumed that the camera is situated at the origin of the object co-ordinate system and the x,y axes of the image plane are aligned with the X,Y axes of the object co-ordinate system, the relationship between image observations (x, y) and the 3-D co-ordinates of the object point (X, Y, Z) can simply be expressed as

$$\begin{cases} x = -c_f \dfrac{X}{Z} \\ y = -c_f \dfrac{Y}{Z} \end{cases}$$

Considering that the initial positions (X_0, Y_0, Z_0) of the targets on the object are known and the current positions are related to the six transformation parameters, equation ? can be extended as

$$\begin{cases} x = -c_f \dfrac{r_{11}X_0 + r_{12}Y_0 + r_{13}Z_0 + x_t}{r_{31}X_0 + r_{32}Y_0 + r_{33}Z_0 + z_t} \\ y = -c_f \dfrac{r_{21}X_0 + r_{22}Y_0 + r_{23}Z_0 + y_t}{r_{31}X_0 + r_{32}Y_0 + r_{33}Z_0 + z_t} \end{cases}$$

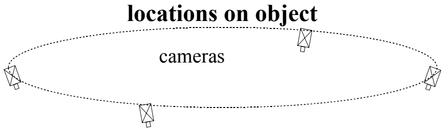
With a minimum of three targets the six transformation parameters can be solved directly by the least squares estimation. The full covariance matrix of the estimates can be obtained easily.

5. Simulation tests

The simulation tests were conducted with a four camera network. A CAD object was generated with nine targets at known positions. The initial positions of the targets are listed in Table 1 and network configuration is illustrated in figure 5.

Targ	X_0(m	Y_0(m	Z_0(m
	10	10	0
	10	-	0
	-	-	0
	-	10	0
	0	10	50
	10	0	50
	0	-	50
	-	0	50
	0	0	50

Table 1. CAD information relating to target locations on object



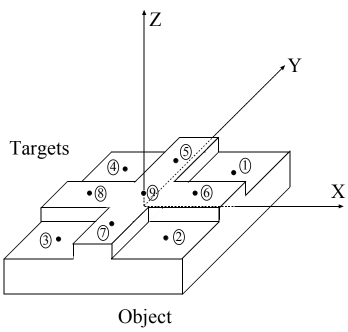


Figure 5. Camera to object relationship

The CAD object was first set to a known position with three translations and three rotations (true value). The image co-ordinates on cameras were computed for the targets on the object. Random errors were added to the image co-ordinates. The six transformation parameters were then calculated based on the image observations and the known 3-D co-ordinates of the targets on the object. The standard deviations of the estimated parameters were also produced. Figure 6 shows an example of the simulation tests (translations are in millimetres and rotations are in degrees).

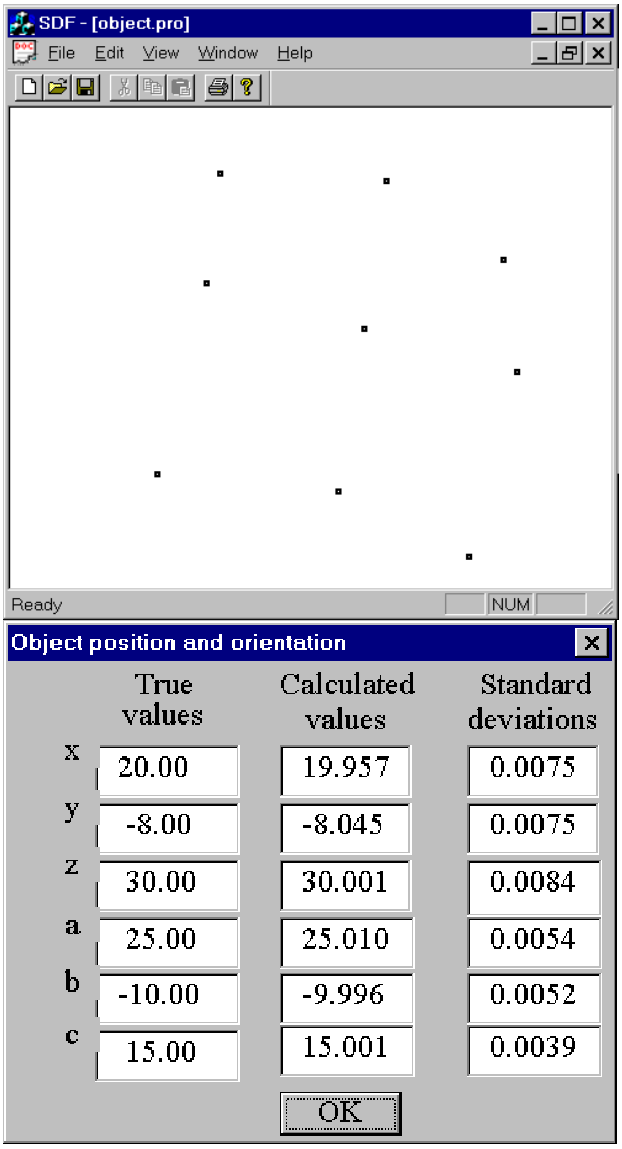


Figure 6. Simulation trial results

To test the measurement capabilities of the six DOF of the CAD object, various changes were made on the measurement system in terms of 2-D subpixel accuracy, convergent angles of the network and the depth of the CAD object. The accuracy of the six DOF were analysed with respect to these factors. The primary test was based on measurement network with the object to camera distance as 1500 mm and the convergent angle as 90 degrees. The 2-D subpixel accuracy was σ_0 = 0.4 μm. The simulation test results are listed in tables 2, 3 and 4.

σ_0(μm)	σ_x(mm)	σ_y(mm)	σ_z(mm)	σ_α(deg)	σ_β(deg)	σ_γ(deg)
0.1	0.0019	0.0019	0.0022	0.0014	0.0014	0.0009
0.2	0.0038	0.0038	0.0043	0.0029	0.0029	0.0018
0.4	0.0077	0.0077	0.0086	0.0057	0.0057	0.0036
0.6	0.0115	0.0115	0.0229	0.0086	0.0086	0.0053
0.8	0.0153	0.0153	0.0172	0.0114	0.0114	0.0071
1.0	0.0191	0.0191	0.0215	0.0143	0.0143	0.0089

Table 2. The influence of 2-D subpixel accuracy on the 6DOF

CV(deg)	σ_x(mm)	σ_y(mm)	σ_z(mm)	σ_α(deg)	σ_β(deg)	σ_γ(deg)
20	0.0079	0.0079	0.0321	0.0116	0.0116	0.0031
40	0.0075	0.0075	0.0174	0.0092	0.0092	0.0032
60	0.0074	0.0074	0.0121	0.0074	0.0074	0.0033
80	0.0075	0.0075	0.0094	0.0062	0.0062	0.0035
100	0.0078	0.0078	0.0080	0.0054	0.0054	0.0037
120	0.0082	0.0082	0.0071	0.0048	0.0048	0.0039
140	0.0086	0.0086	0.0065	0.0045	0.0045	0.0041
160	0.0089	0.0089	0.0063	0.0043	0.0043	0.0043
180	0.0090	0.0090	0.0062	0.0043	0.0043	0.0044

Table 3. The influences of the network convergent angles on the 6DOF

depth(mm)	σ_x(mm)	σ_y(mm)	σ_z(mm)	σ_α(deg)	σ_β(deg)	σ_γ(deg)
0	0.0072	0.0072	0.0088	0.0062	0.0062	0.0036
20	0.0073	0.0073	0.0087	0.0061	0.0061	0.0036
40	0.0075	0.0075	0.0087	0.0059	0.0059	0.0036
60	0.0078	0.0078	0.0086	0.0056	0.0056	0.0035
80	0.0082	0.0082	0.0085	0.0052	0.0052	0.0035
100	0.0085	0.0085	0.0084	0.0048	0.0048	0.0035

Table 4. The influence of the depth of the object on the 6DOF

The 2-D subpixel accuracy σ_0 acts as a linear factor influencing the accuracy of the 6DOF. This is clearly shown in Table (). The increase of the network convergent angle improves the accuracy of the positional parameter z_t and the rotational parameters α and β significantly but decreases the accuracy of the rotational parameter γ. The best convergent angle for the accuracy of the positional parameters x_t and y_t is about 60 degrees. However as far as the RMS standard deviations of the three positional parameters and the three rotational parameters is concern, the best convergent angle for the positional parameters is at 110 degrees and the best convergent angle for the rotational parameters is around 160 degrees. Therefore a convergent angle between 110 and 160 degrees will give the best estimation of the six DOF provided that the 2-D subpixel accuracy σ_0 remains the same. An object with depth can improve the accuracy of the three rotational angles but will not help the positional parameters.

6. Conclusions

The method described here allows an easy analysis for the capabilities of the measurement system with the covariance of the estimates.

Further work

7. References