A survey and examination of subpixel measurement techniques.

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ABSTRACT.

There is increasing use of electronic sensors and digital signal processing for measurements of optically acquired data. Applications include automatic inspection, surveying, remote sensing and photogrammetry. Sensors, at the present time, require subpixel methods to improve the resolution above that available given the spacing of sensing elements and the analogue to digital conversion resolution. This paper reviews proposed subpixel methods in the context of an increasingly important application, namely, the determination of the position of a laser spot on a sensing array for triangulation. A number of techniques are chosen and analysed experimentally. Their performances are compared and contrasted with respect to spatial resolution, quantisation accuracy and noise. For the comparison, use is made of simulated data, and real data obtained from a triangulation system[9].

1. INTRODUCTION.

Electronic sensors and digital signal processing are increasingly being utilised for measurements of optically acquired data. Applications where measurements are made include automatic inspection, surveying, remote sensing and photogrammetry. Both 1-D and 2-D sensors are, at the present time, of relatively low resolution (2000:1) compared to more traditional devices such as photographic film (100000:1). Much time has been devoted to improving the performance of these sensors by using subpixel algorithms to improve the resolution above that available given the spacing of sensing elements and quantisation by the analogue to digital conversion. This paper reviews proposed subpixel methods in the context of an increasingly important application, namely the determination of the position of a laser spot on a sensing array for triangulation. Subpixel techniques considered include centroiding, interpolation, correlation and edge detection. A number of techniques are analysed experimentally namely centroid, weighted centroid, vernier and interpolation. Their performances are compared and contrasted with respect to spatial resolution, quantisation accuracy and noise. For the comparison, use is made of simulated data as well as real data obtained from a triangulation system. The real data was acquired from a test apparatus configured to verify accuracy of subpixel resolution for the determination of the position of a laser spot along a 1-D solid state array.

Subpixel techniques have been used in many areas of application such as: star tracking, satellite and space probe imaging and in measuring dimensions of products such as rolled steel strip and manufactured components i.e. position of edges and objects. In each application, knowledge of the physical nature of the scene imaged onto the sensor is used e.g. the edge of steel sheet may be considered a unit step. This is modified by the electronics, optics, lighting, surface reflectivity, diffraction etc. to form a sampled intensity map of the edge from which the position of the edge can be recovered. Objects such as stars and laser spots may have a known shape e.g. Gaussian, allowing subpixel registration with the model by computation of the position of the centroid or peak position after interpolation.

2. IMAGE FORMATION.

Subpixel accuracy can be achieved because the imaging process can be accurately modelled. The *a priori* knowledge of an object intensity distribution, such as a star (a point source), is essential for obtaining subpixel accuracy, and enabling a decision as to whether the data is valid for subpixel treatment. However for a full understanding of the data as recorded by the imaging system, further knowledge of the characteristics of the optics, sensor and signal conditioning electronics is required.

The case of an optical triangulation system will be looked at to provide an example of obtaining subpixel accuracy and the importance of understanding the imaging system.

2.1. Optics.

The optical triangulation distance measurement technique requires a small collimated beam of light (usually a laser beam) to identify a point on a structure, the reflected light of which is processed by the lens system to produce an modified image on the sensing element (often a CCD sensor). The analysis shows that if the Fresnel integral[27] is solved it is found that[3]: "a Gaussian Source Distribution remains Gaussian at every point along its path of propagation through the optical system". Hence, in the special case of laser generated object spots imaged by a lens onto a sensor, it is appropriate to take a Gaussian intensity distribution as the model for subpixel image registration techniques.

2.2. Sensor.

Ideally the sensor will sample the data at discrete points i.e. using delta functions. In which case the modulation transfer function (MTF)[16] will have a uniform response up to the Nyquist frequency which determines the maximum spacial frequency resolvable. However, in practice, the sensing elements have finite size and are usually rectangular in shape. Hence there is a low pass filtering effect which is manifest in the MTF as a falling response at high spatial frequencies. High frequency response is also affected by deep carrier crosstalk[19] which occurs for infra red light. Both of these properties reduce the spacial bandwidth of the sensor but are not critical for this application because of the laser frequency chosen, and because the Gaussian spot occurs over a significant number of pixels.

Other sources of error are due to dark current, photo-response non- uniformity and poor charge transfer efficiency. In modern sensors these are relatively small and unimportant.

2.3. Data processing.

The spacially sampled analogue signal from the sensor is processed to produce a stream of digital information. Noise is generated by quantisation (in A/D conversion), amplification, black level clamping and signal restitution. Of these, quantisation noise is the most important especially as most sensor arrays have a higher dynamic range than the typical A/D converter used (8 bits).

3. SURVEY OF PROPOSED TECHNIQUES.

The desire for some method of obtaining subpixel accuracy has occurred in many disciplines including photogrammetry, automatic inspection, remote sensing and computer vision. A large number of techniques have been proposed that fall into a number of categories. These are centroiding, interpolation, correlation, edge analysis and shape-based. Most techniques fall into one of these categories. However, in some cases, more than one technique are used in combination.

Subpixel accuracy is necessary because the data is a digital representation of an analogue signal that has been sampled onto a discrete array (usually 1-D or 2-D rectangular) and each data point quantised to a finite number of levels (usually 8 to 12 bits). In some cases the sampling onto an array is achieved by the sensor used. For instance a solid state array consists of a discrete grid of samples. The size of the arrays restricts the number of samples that can be used to typically 2048 (1-D array) and 512x512 (2-D array). To obtain the required measurement accuracy it is necessary to somehow determine measurements to points between the sample positions.

Apart from the quantisation errors other factors need to be accommodated including noise and the system response of the lens and array as well as other application dependent factors such as speckle.

Although we are concerned in this paper with techniques for 1-D subpixel measurement, 2-D techniques are considered as much research effort has gone into measurement from images. In addition, in our 1-D application, a single shape of object is considered that approximates a Gaussian. However in the 2-D case different shapes or features are considered ranging from spots, crosses, circles through to edges and combinations of edges. It is true to say though that these techniques may be applicable to 1-D use.

3.1. Centroiding.

Many techniques have been proposed that are based on determining the centroid of an isolated object in the image or waveform. Given a symmetric object, the centroid will give a perfect result with ideal data. The techniques differ in the way the centroid is computed and the type of data used.

Some of the earliest examples are in star trackers for satellite positioning[4,35,24] in which a star is imaged onto several adjacent sensor elements as an unimodal intensity distribution. In one example[4] the star of interest is Polaris. The centroid is computed by analysing the imbalances of intensities in the sensor elements using the knowledge that the intensity distribution in symmetric. In other papers on star trackers, the accuracy is quoted as 0.0625 pixels[35] and 0.05 pixels[24].

The standard first order moment (centre of gravity) is used to accurately locate matching points for a stereo algorithm[18]. The grey level pixel values are used and an accuracy of +/-0.4 pixels stated. A two stage process has been proposed for accurately locating targets in a grey scale image[12]. The first stage assumes a solid blob on a contrasting background with the centroid computed from the blob area using the standard first order moment. A second stage is used if there is a clearly marked centre to the blob (white spot on a dark blob). In this the grey levels in a window (3x3 or 5x5) around the centroid are used to compute the centroid. Accuracy was reported as better than 0.1 pixels.

An analysis of centroiding[10] considers the effect on the accuracy of sampling rate, noise variance, quantisation resolution and signal to noise ratio. Simulated data is used and it is shown that accuracy is mainly affected by signal to noise ratio and quantisation resolution. For the typical case of 8 bit quantisation (256 levels), an RMS error in position is less than 0.06 pixels for a signal to noise ratio of 10.0. Also considered is the affect of thresholding the data to reduce the number of pixels processed. High signal to noise ratios allow a threshold to be used without significantly reducing the accuracy.

A modification to the standard centroid calculation[38] weights pixel values by effectively squaring them. This reduces the effect of small pixel values at large distances from the centroid to reduce the effect of asymmetric objects. In addition a threshold is used to reduce the number of pixels considered. A thorough analysis of this algorithm is presented for various values of quantisation resolution, signal to noise ratio and asymmetry of the target. It is concluded that an accuracy of 0.01 pixels can be obtained under ideal circumstances. Below 5 bits/pixel quantisation and signal to noise ratio of 5:1 the accuracy deteriorates. In addition, the affect of asymmetry can be reduced by a correct choice of threshold.

A novel method of computing the centroid of a quasi-symmetric signal uses a recursive spectral phase algorithm[25]. In essence the algorithm computes the position along the waveform where the imaginary components of the Fourier spectrum are a minimum i.e. the antisymmetric part is minimised. For 1-D signals the accuracy is stated as being of the order of 1 pixel. The effect of Gaussian noise with a signal to noise ratio of 2 resulted in the uncertainty on centroid position defined by a Gaussian random variable with variance 0.2 pixels.

3.2. Interpolation.

Interpolation has been used to obtain subpixel accuracy. In this technique methods are used to generate grey level values between the discrete samples which can then be searched for positions of features at subpixel resolution such as the peak of a Gaussian or maximum gradient for an edge. The classical technique is to use a polynomial fit which has limitations[20] such as computational speed and choice of order of polynomial. A high enough order is required to fit to all the data points with low error. This has been extended[37] to deal with irregular sampling intervals and the amount of computation required. Other polynomial techniques proposed include hypersurface approximation[23].

Another technique is based on convolving the sampled data with various formulations of low pass filter. This technique is dependent on the original analogue signal being sampled according to Nyquist. Then convolution with a sinc function will recover the original signal. However the sinc function has infinite extent and is impractical to implement unless truncated which can then produce errors. Instead approximations to the sinc function can be used such as B-splines[5]. Other techniques have been suggested: frequency domain analysis[6], generalised splines[7,8], V-splines[2] and a 2-D triangulation technique[26].

Multiple images have been used to obtain subpixel accuracy. Several low resolution images with relative subpixel displacements are combined[34]. A high resolution image is estimated and iteratively improved by considering the errors between the low resolution images and those derived from the high resolution estimate. To improve linear resolution by a factor of 4, 16 images are required. However, using a lower number of images still produces good results. Jittering[21] is used to acquire two low resolution images with one translated by 0.5 pixels. These are then combined together to produce one high resolution image of twice the resolution. Improvement in the resolution can be obtained by using more images at equal translations. The reduction in effective pixel spacing is then defined as *1/n* where *n* is the number images.

3.3. Correlation.

Correlation has been used to obtain subpixel accuracy mainly for the determination of the position of a known target in an image to determine the translation, rotation and scale parameters. In some cases the target is obtained from another image. An example of an application is matching between aerial photographs[13]. Correlation is considered because the calculated cross correlation function is smooth enabling interpolation to find the correlation peak. An analysis of the technique[36] considers translation only stating that the maximum error should be no more that 0.5 pixels for the a signal to noise ratio of 1 (random noise). With less noise the resolution will be more. A correlation algorithm is proposed[32] for measuring the displacement of speckle patterns. Correlation is performed first followed by interpolation using a least mean square third order polynomial technique to find the peak. The accuracy is quoted as to within 1 micron for a pixel spacing of 13 microns i.e. 0.08 pixels.

3.4. Edge analysis.

Methods have been proposed that determine the location of edges to subpixel accuracy. The shape of the edge is usually chosen to be an ideal step edge or one that has a rounded shape because of low-pass filtering by the sensor and lens. A second order differential operator[28] produces a zero crossing at what is normally defined as the position of the edge (where a point of inflection occurs). The zero crossing can be detected at subpixel accuracy. This has been used for stereo matching and measurement[28]. Problems with this technique include the choice of the width of the operator to detect the required edges and the affect on accuracy of edges in close proximity. Edge positions are corrupted (pushed apart) if two edges are close compared with the width of the operator. Interpolation has been used[30] to obtain a high resolution image after which edge detection is performed. The increase in resolution allows smaller operators to detect edges with increased accuracy because of the reduction in interaction between close edges. A similar technique has been proposed[31] in which an interactive method is used to obtain the position of edges to subpixel accuracy. This is based on the principle that for a particular known filter used to digitise the analogue data, an optimum reconstruction filter is available. This can be used in combination with a second order operator to detect the position of an edge in a particular orientation. From an initial estimate of edge position and orientation, increasingly accurate edge position estimates are obtained by iteration. Accuracy quoted for the measurement of the diameter of a steel ball occupying 50% of the image is 0.03 pixels.

Accurate location of edges can be performed using moments[39]. It is shown that the first three moments can be used to determine the three degrees of freedom describing a 1-D edge (edge position, contrast and background level). The moments can be computed from the grey level pixel values and used to determine the subpixel position of the edge. A 2-D version is also proposed with the addition of edge orientation and the performance of both methods extensively analysed in terms of noise, quantisation effects and various edge shapes. For real image data, an accuracy of better than 0.05 pixels is stated.

3.5. Shape-based.

All of the above techniques can be configured for 1-D and 2-D image data. However there are techniques that can be used on 2-D images given that knowledge of the shape of the feature being measured is available e.g. a straight or circular edge. For binary edges, subpixel accuracy can be obtained by fitting a line to the binary edge data[14]. Note accuracy is reduced for edges that are vertical, horizontal and at 45 degrees. Other techniques have been proposed for straight lines[11,22],circles[29] and crosses[1]. The accuracy of measuring such features has been analysed theoretically[17] for binary images.

4. PERFORMANCE DETERMINATION.

To determine which method out of the number cited above is best suited to a particular application requires one of a number of analyses. Ideally a theoretical analysis is used. Havelock[15] has presented a theoretical analysis of the problem of subpixel accuracy for the determination of the centroid of a 2-D object after sampling and quantisation. He mainly considered noise free cases but did present some results on the effect of noise on the results. A theoretical comparison of four methods used for subpixel registration[40] shows which is the best method for measurement of object movement from the comparision of pairs of speckle images. Intensity interpolation followed by correlation gives an accuracy of 0.01 to 0.05 pixels for real speckle data. An analysis has been presented[38] for a particular type of centroiding algorithm based on the use of simulated data.

In this paper experimental analysis is used on both simulated and real data pertinent to optical triangulation. Although meaningful results can be obtained for such effects as quantisation, noise and spacial resolution, it is not possible to directly translate the subpixel accuracy to measurement accuracy because to do so would ignore laser pointing stability, environmental effects, speckle, surface irregularities in form and contrast, and mechanical instabilities. However, if ideal image location error is known then a comparison between ideal accuracy and that achieved in practice is possible.

To investigate the effects of quantisation, spacial resolution and noise on the accuracy of measurement, simulated data was generated. Three methods of accurately measuring the location of the laser spot were investigated namely centroid[10], weighted centroid[38] and vernier[33]. These were chosen as they are relatively simple to implement in this 1-D application and are well suited to subpixel location of an approximately Gaussian spot.

4.1. Use of simulated data.

It has already been stated that the intensity distribution of the laser spot on the sensor is approximately Gaussian. The major reason for departure from a true Gaussian is the presence of speckle and surface irregularities. In this analysis these were ignored.

Simulated data was generated from the equation of a Gaussian distribution as a real continuous function. This was then sampled, quantised and corrupted by noise of various amounts. The Gaussian function was translated by subpixel amounts (1000 points over one pixel spacing) before digitisation. Then each of the three methods were used to compute the location of the spot for the 1000 instances at various quantisation levels, signal to noise ratios and spacial resolutions. Figure 1 shows graphically the results for each method. The graphs show the standard deviation of the errors obtained for the 1000 trials. Analysis of the distribution of the errors for all methods indicates approximately Gaussian distributions. As expected, the more quantisation levels or the greater the signal to noise ratio, the more accurate the results. An observation is that noise and quantisation effects are approximately equivalent i.e. the effect of halving the number of quantisation levels can be offset by doubling the signal to noise ratio.

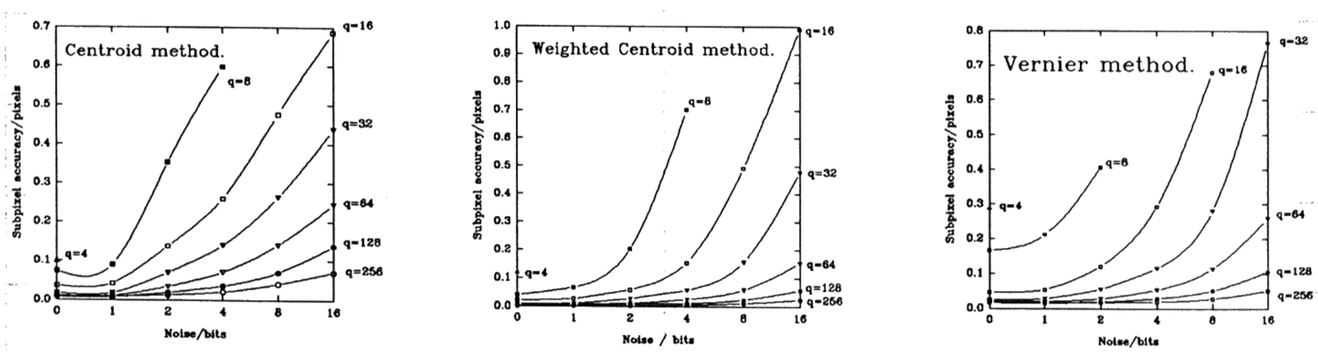

Figure 1. Results of measurement of subpixel accuracy over 1000 trials for three different algorithms. Each curve represents constant number of bits for the amplitude of the Gaussian. The x axis indicates the amplitude of the noise in bits and the y axis represents the accuracy in pixels.

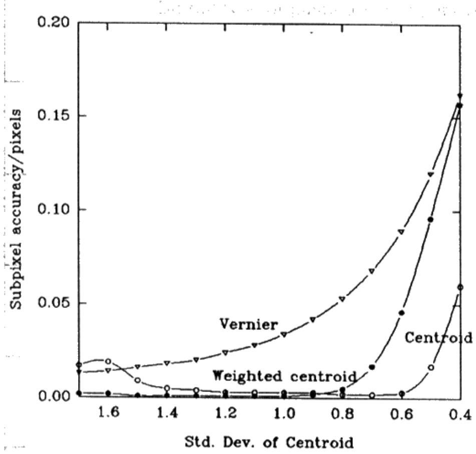

Figure 2. Effect on accuracy of the width of the Gaussian.

Figure 2, shows the result of changing the width of the Gaussian spot (equivalent to changing the spacial resolution of the sensor). The x-axis indicates the standard deviation of the Gaussian spot with a value of 1.0 meaning the spot was spread across approximately 6 pixels. For all three methods used for this experiment, the accuracy improved as the resolution increased. For the two centroiding methods, the accuracy quickly reaches a minimum as the resolution is increased. At low resolutions all methods gave poor results. The weighed centroid was worse at low resolutions because the spot was only represented by at most 3 pixels and the outer two pixels had little effect as they were of much lower amplitude than the centre pixel. As each pixel value was weighted by the itself the centre pixel had a disproportionate effect.

4.2. Use of real data from the experimental triangulation system.

To determine the accuracy obtainable from a triangulation system[9] used with subpixel measurement algorithms, an experimental method was devised. The target was a diffuse reflecting spray painted surface which was mounted on the rear of a retro-reflecting prism. The prism was used with an interferometer to check that no movement took place between the laser source and the target. The environmental conditions were such that the temperature of the air and apparatus were stable and there was little air movement. Eight sets of data were collected, each consisting of 1000 trials. Each trial consisted of the acquiring the image data and the accurate position of the target from the interferometer. Two of these sets were analysed for this paper. For data set 7, the laser and camera were mounted 3 metres from the target, and for data set 8, the laser was moved to be 150 mm. from the target (the camera was kept in the same position). Moving the laser was to ascertain if errors occurred because of laser pointing stability and environmental effects. Unlike the simulated data, the spot was not moved for these trials but left stationary. This is to remove the effect of changes in the speckle pattern on the results as it would be a stationary pattern. In fact the speckle had little effect on the data. The errors in the results would then be related to sampling resolution, quantisation and noise.

Method / Data set 7.	Standard deviation / mm
Centroid	0.062
Weighted centroid	0.068
Vernier	0.052
Interpolation	0.074

Method / Data set 8.	Standard deviation / mm
Centroid	0.042
Weighted centroid	0.043
Vernier	0.017
Interpolation	0.042

Figure 3. Results of subpixel accuracy assessment over 1000 trials for 4 different methods.

Figure 3, shows the results for the four methods for the two data sets. In addition to the three techniques already investigated, interpolation[5] was also used. For both, the system produces subpixel accuracy. The best method for both data sets was the vernier method with the other three giving similar results. There is obviously an effect associated with the laser as the subpixel accuracy is higher when the laser is closest to the target.

5. CONCLUSION.

This paper has discussed methods that could be considered for obtaining subpixel measurements for triangulation systems based on laser light sources and solid state 1-D sensors. Although there have been many methods proposed for subpixel measurements, many are not suitable to this 1-D application. There are five categories of technique of which centroiding, interpolation and correlation are the most applicable to this application area. Most techniques were capable of obtaining an accuracy better than 0.1 pixel, and in some cases better than 0.05 pixels. Three methods were chosen from the literature and compared using both simulated and real data. Subpixel accuracy was achieved for each of the methods with accuracy specified statistically as the standard deviation of the error between the actual position and the measured position. Further work is required on which is the best method as the weighted centroid produced the best results in simulation but the vernier was better on real data.

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