An analysis of subpixel target location accuracy using Fourier Transform based models

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ABSTRACT.

The simulation of the performance of subpixel algorithms under known conditions is a valuable tool. For instance, the optimum size for a target image can be determined as well as the influence of varying the threshold level or target size. Previous work (West & Clarke, 1990; Trinder, 1989) looked at the effects of quantization and additive noise on the location of target images. More recently, work by Shortis et al, 1994 has tested more algorithms and looked at other effects such as those caused by saturation and DC offset. In this paper the electronic noise present in imagery from typical CCD cameras is both measured and used within a simulation of the subpixel location performance of the centroid and squared centroid method. The effect on target location of uneven background characteristics are also analysed. To achieve this the physical characteristics are modelling by using the Fourier Transform of both background and target models and combining them in the frequency domain. By performing the inverse Fourier Transform the resulting target image is used to locate the target. The simulation methodology will be explained and tests performed so that a better understanding of the factors that contribute to subpixel errors can be gained.

Keywords: subpixel precision, targets, photogrammetry, Fourier Transform, noise, simulation.

1. INTRODUCTION.

In this paper the first section deals with the construction of appropriate models for target image formation and a simple analysis of noise encountered in CCD/frame-grabber combinations. The second section extends previous target simulation work at City University to include better target models and noise. In the third section Fourier Transform methods are used to model the effect of various backgrounds on computed target location errors. Finally, some laboratory based test results are compared with the simulation and Fourier Transform based analysis.

2. TARGET IMAGE FORMATION AND TARGET MODELLING.

2.1 Estimation of the noise levels from frame grabbers.

To ensure that the simulation experiments were related to real CCD camera/frame-grabber systems the noise experienced in these systems was measured. A ground glass screen was placed in front of a TM6CN CCD camera and given various levels of illumination from a distributed light source. A 50x50 pixel square section of the image was grabbed and analysed. The standard deviation of the resulting levels of noise were then computed and are illustrated in figure 1.

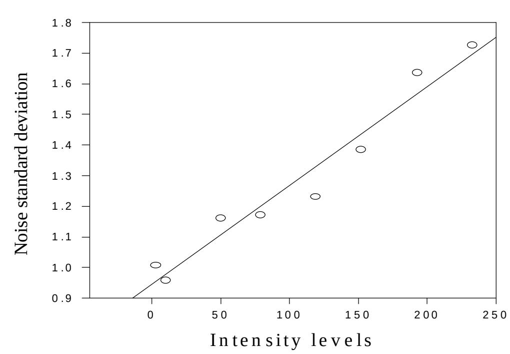


Figure 1. Standard deviation of electronic noise at varying intensity levels.

These results illustrate that the electronic noise levels are considerable when compared to quantization noise. A maximum difference between grey levels at a mean of about 230 intensity levels was 13 grey levels. Furthermore, it appears that with the camera used that noise increased with the mean intensity level of the background. These measurements were taken on a hot day in the Summer with a camera which had been switched on for some time. It is well known that the level of electronic noise is related to temperature, for example dark current noise is doubled for approximately every eight degrees increase in temperature. In astronomy, and other low light level applications such as microscopy, cooling is used to obtain good noise immunity and long exposure times. To investigate this subject further a camera was placed in a freezer and cooled to -3°C. The noise levels were then measured from switch on to temperature stabilisation. The results are illustrated in figure 2, and confirm the expected increase in noise with temperature. These results are also generally comparable with the results obtained by Beyer, 1992. Further investigations are warranted because these noise levels appear to be slightly higher than those reported by Beyer. These would probably require the accurate measurement of the temperature of the sensor and comparisons with other cameras, and so were not considered necessary for these investigations as only the general level of noise that is expected in practise is necessary for the simulation trials.

Before it is possible to use the information from these tests it was necessary to check that the noise could be modelled by random noise with a Gaussian distribution. For one of the 50x50 sets a histogram of the intensity distribution was plotted (Figure 3). This does show that a Gaussian distribution is a reasonable approximation to practise, with only a small difference further from the mean where more values than would be expected were observed. This was confirmed by checking the distribution of a randomly generated set of data where the maximum and minimum values were inside the values obtained in practise. This slight difference may be explained by other noise effects which are superimposed on the electronic noise.

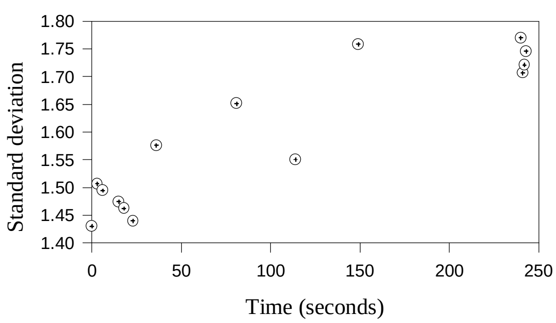


Figure 2. Graph of electronic noise against time.

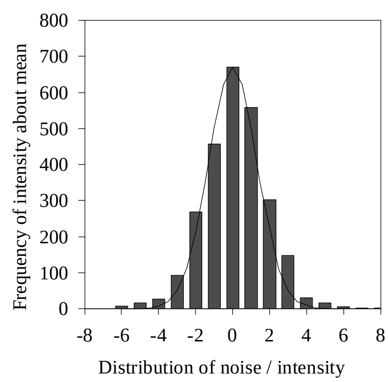


Figure 3. Histogram of intensity distributions.

Having confirmed the general nature of the random noise a computer program was written to create such noise in the simulation program. To simulate the frame-grabber/camera noise a random number generator algorithm (Press, 1992) was used which generates noise with a Normal Distribution. The noise generated by this algorithm was checked by plotting its histogram and comparing the distribution with an ideal distribution and was found to be acceptable to use in practise. Care was taken to ensure the seed for the random number generator was changed for each computation to ensure a different set of random numbers for each simulation test. Quantization noise is used in the simulations to indicate the best case results that can be achieved with an A-D converter of a given range. However, these effects are relatively small compared to those arising from the frame-grabber/camera combination. To simulate quantization noise the ideal target image intensity is rounded into an integer.

2.2 The simulation model.

In past simulations conducted by Clarke (West and Clarke, 1990; Clarke, et al., 1993; Shortis, et al. 1994) a Gaussian shaped simulation of a target image was used. However, while this may simulate correctly the image formed of laser targets (often the case with optical triangulation systems) and small retro-reflective targets, such a model does not correctly model the image formed of a large retro-reflective target. When the target is relatively small, the image formed is essentially the Point Spread Function of the lens, which is approximately Gaussian in shape. To improve the simulation a model is required which reflects the performance of the lens. It was thought that a Gaussian shape would still model this properly provided that a single standard deviation was used to reflect a given lens characteristic and the size of the target modified by introducing a peak intensity height circle and making the edge of the circle a Gaussian shape. To investigate this a number of targets of varying size were imaged and compared to the hybrid Gaussian function. Four of the target images are illustrated in figure 4. The lens used was manufactured by Computar was fixed focus (6.5mm.). The targets tested were 11.5, 5, 4, 3, 2.5, 2, 2.5, 2, and 1.5 mm. in diameter and were placed one metre from the lens. The aperture was varied and the images collected at differing spatial locations so they were as symmetric as possible to aid comparison with the synthetic target images. The aperture was varied to obtain images of approximately equal intensity. Previous tests had shown that varying the aperture made little difference to the spatial frequencies resolved by the lens as it appeared that lens aberrations effects dominated over the effect of diffraction (wider aperture = sharper image).

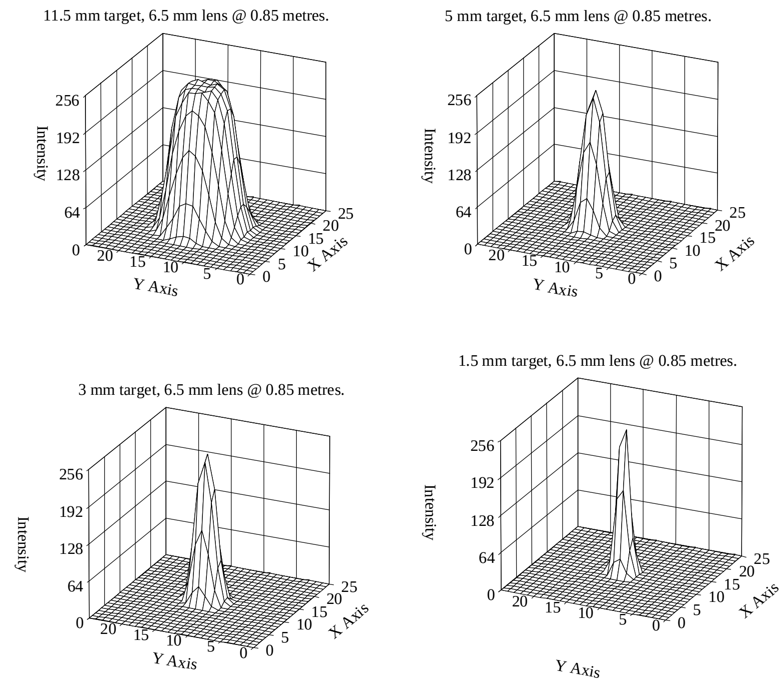


Figure 4. 3-D views of the 11.5, 5, 3, and 1.5 mm. target images.

These targets were then compared to those generated by the hybrid Gaussian function and are illustrated for a few of the cases in figure 5. Various sigma values were used to obtain the best fit between images and models. A sigma for the Gaussian shape of 1.2 was found to be optimum for the lens used. It is clear that this model is close to the physical reality and so this model was then used in a new simulation.

2.3 The Fourier Transform model.

In addition to improving the target simulation and adding noise the effects of background on target image location are also studied using Fourier Transforms. Various approaches to modelling the lens and CCD sensor combination using the Fourier Transform methods were considered. The use of a low-pass filter in the frequency domain gave reasonable results but was not considered an ideal model of the effect of the lens in the system. Hence, further tests were conducted to construct an Optical Transfer Function (OTF). The point spread function of a lens is an approximately Gaussian shape which can be demonstrated by producing an image in response to a impulse input (i.e. a vary small but bright spot of light).

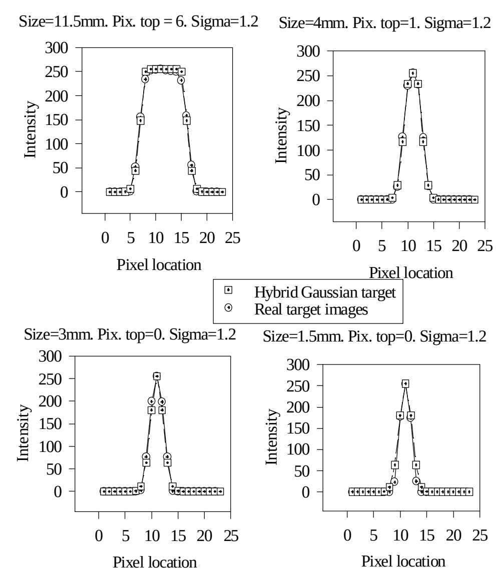


Figure 5. Comparison between hybrid Gaussian and

real target image cross sections.

The tests conducted for the Gaussian image simulation confirm this. The sigma measured in the previous tests was used to produce the correct size Gaussian function to model the lenses used in these tests. Various size simulated target objects were tested to show that all target images were comparable to similar sized real target images. Figure 6. compares the real images with the Fourier Transform images. The figure illustrates that the target images from the Fourier Transform are very similar to those for a real image. Again the available factors have been adjusted to produce this result. The parameters used in generating these targets were then used in subsequent tests.

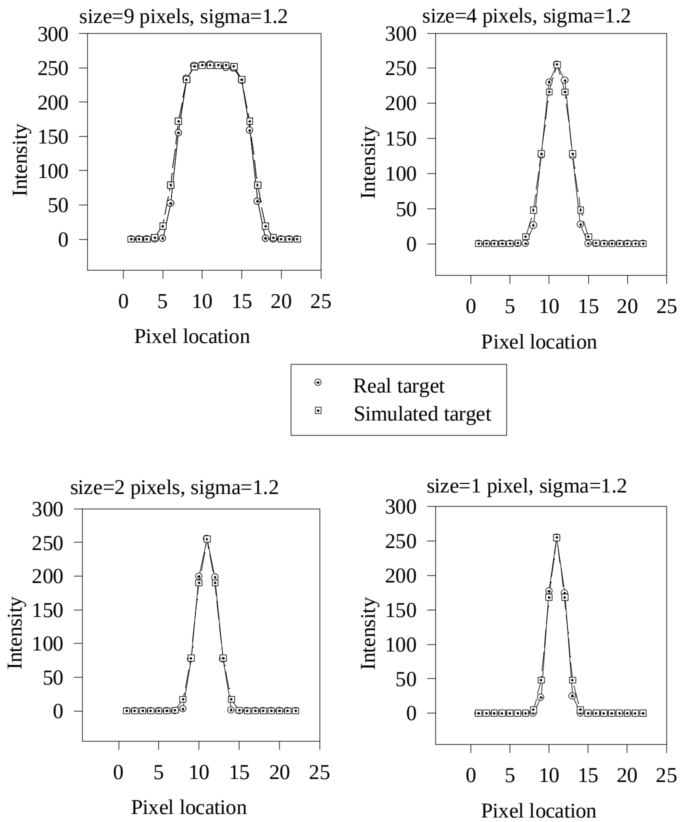


Figure 6. Comparison of real and Fourier Transform target images.

3. SIMULATION RESULTS.

To investigate the effect of electronic noise in addition to quantization noise the centroid and squared centroid methods were chosen. Three sets of tests were conducted to measure the error in location due to: noise added to a typical target image; the variation in the size of the target image; and the effect of varying the threshold for a typical target image. In each case 2500 target locations were computed for an 50x50 array of subpixel locations within a 1x1 pixel grid. In addition, the effect of DC offset in the frame grabber was considered as well as the option of not using a threshold in the centroid computations. The effect of DC offset in the production of subpixel target location errors has been discussed by Clarke, 1995. The majority of frame grabbers will be set up so that the noise level at zero light input will produce a zero signal level. This implies that the mean level of the video signal is at least three times the standard deviation of the noise below the point at which the A-D converter produces a ‘one’. In this simulation the noise has been added to the signal so that the noise is above the zero threshold of the A-D converter. This allows either a threshold to be used to cut out the noise from the centroid computation or no threshold to be used so that background noise is included. To simulate the typical frame-grabber set up, and option to subtract a DC offset from the simulated target image was also included and tested. While this will mean that the signal to noise level that is used in each of the computations will not be identical in each case, it would appear that the DC offset configuration is close to reality and the other cases are close enough to each other to be comparable.

3.1 The effect of varying the size of a target image on target location error.

The size of the target image was varied as discussed in the previous section by using a Gaussian shaped target of standard deviation 1.2 and varying the size of a central radius circle from zero to four pixels. This implies a range of target size from approximately seven pixels in diameter to fifteen pixels. Figure 7(a) illustrates that with a fixed threshold of 12 grey levels the squared centroid method initially improves in location accuracy before beginning to worsen, while the centroid method gradually improves. This may be explained by considering the effect of squaring the intensity levels, this gives better accuracy to start with due to the concentration on the higher intensity levels, but as the target gets bigger the higher pixels are all of the same intensity and so with the addition of noise the error increases. However, for the centroid method the computation is not so biased to the higher intensity values and so as the target gets bigger there are more edge pixels and so the location accuracy improves. It should be noted that the cross over point does not happen for these algorithms until the target is larger than fifteen pixels in diameter. For the results illustrated in figure 7(b), where the background intensity has been included, then the error in location for the squared centroid method is a little larger than for the threshold of twelve grey levels and a similar trend may be noted. The results for the centroid method are worse by a factor of four. This may be explained by considering the relative importance of the lower intensity levels to either method. The higher importance of the noisy peripheral pixels has a significantly adverse effect, the squared centroid method is only marginally worse. No explanation for the peak error in the centroid computation between a size of 0.4 and 0.6 radius can be offered although the algorithm and the target images were carefully checked. Figure 7(c) illustrates the most realistic case for frame grabber/camera combinations where optimum signal to noise has been achieved with retro-reflective targets. For the squared centroid method the results are broadly similar to the threshold of twelve and no DC offset. However, the behaviour of the centroid method is markedly different - it is initially twice as accurate and rapidly becomes as good as the squared centroid method. However, after a radius of about one the error begins to get slightly larger. It is probably wise not to read too much into these results as variations in results are possible between tests when small changes in the threshold level are made. Hence, at this stage it has been concluded that the differences between figure 7(a) and figure 7(b) are not very significant except to say that under these circumstances both methods give excellent subpixel accuracy and are able to location targets, even with the presence of electronic noise, to better than about one fiftieth of a pixel. This level of precision is considerably better than the overall level of precision which may be inferred from using these observations in a multi-camera bundle adjustment. In conclusion these results confirm, what has been noted many times before, that there is no significant benefit to be gained by using targets any bigger than five to seven pixels in diameter.

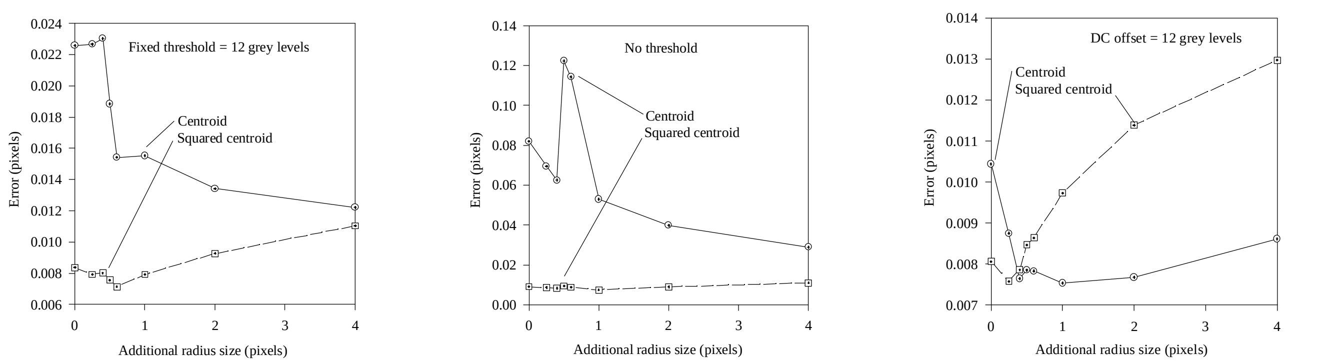


Figure 7(a) Graph of the variation of error with target size (fixed threshold).

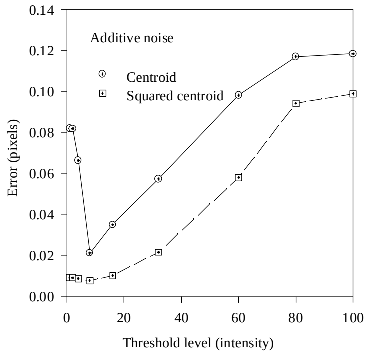
Figure 7(b) Graph of the variation of error with target size (no threshold).

Figure 7(c) Graph of the variation of error with target size (DC offset).

3.2. The effect of varying the threshold level on target location errors.

Figure 8(a) illustrates the effect of varying the threshold level on target location errors for a typical size target image (std. dev. 1.2). The centroid method is initially poor due to the noise of the background which the squared centroid is less affected by. When the threshold level is high enough to remove the effect of the background noise the location of both methods improves. The change is more dramatic for the centroid method than for the squared centroid. As the threshold is increased the errors in location also increase due to the fact that the target is becoming smaller and there is less information for the algorithm to use. It should be noted that in the same way as a DC offset can produce systematic effects (Clarke, 1995) the application of a threshold also has this effect. At certain subpixel locations with a non-symmetric target image the side pixels will be above the threshold on one side of the target image and below it on the other side. This produces a systematic bias in the location that when plotted is often sinusoidal in nature. The squared centroid method is less sensitive to this effect except when the target image is small in size due a high threshold. Tests were also conducted for the effect of threshold variations when a DC offset is used (figure 8(b)). The results do not have the initial errors due to noise in the background, the centroid is reasonably comparable with the squared centroid method, and the errors are larger than for the previous case at higher threshold levels.

This is probably explained by differences in the overall intensity of the target image with the 12 grey levels subtracted from all of the target image.



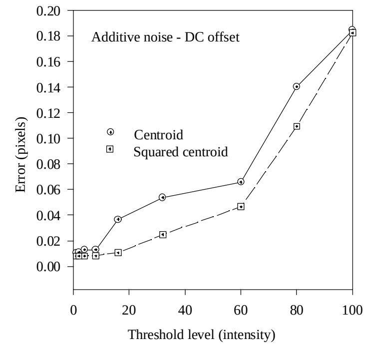
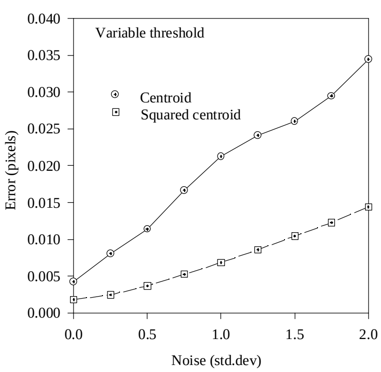


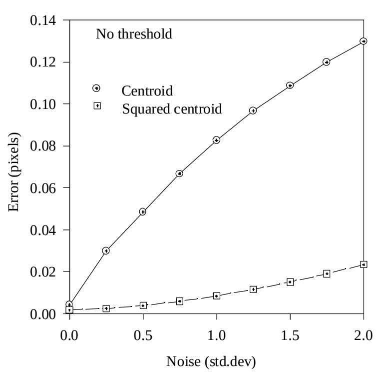
Figure 8(a) Variation in target location error with change in threshold (additive noise).

Figure 8(b) Variation in target location error with change in threshold (additive noise + DC offset).

3.3. The effect of electronic noise in a target image on target location error.

The final tests consider the effect of electronic noise on target location error. For the first case considered which is illustrated in figure 9(a) a threshold was chosen which varied so that it was always just above the noise level. In this case the centroid location method shows an change in error that is increasing at a faster rate than the squared centroid location method as the noise level is gradually increased. It should be noted that for the squared centroid method the target location accuracy is around one hundredth of a pixel for realistic noise levels. Figure 9(b) illustrates the effect of including the background noise in the location computation. As seen previously the centroid method is affected much more than the squared centroid level to the extent that a subpixel location accuracy of on tenth of a pixel is likely to be obtained in this configuration. Finally, the case of the DC offset was considered where the level of the offset was the same as used for the threshold in the tests illustrated in figure 9(a). In this case the results for the centroid and squared centroid both show an increase in error of location with noise. The level of the noise is four times what might be expected from quantization alone but this is still around a hundredth of a pixel. Considering that realistic noise levels have been added to the target images this is an excellent result. It remains for further research, which is currently being conducted at City University into better functional models for the popular small format CCD camera and cheap lens combinations, to make use of this resolution.





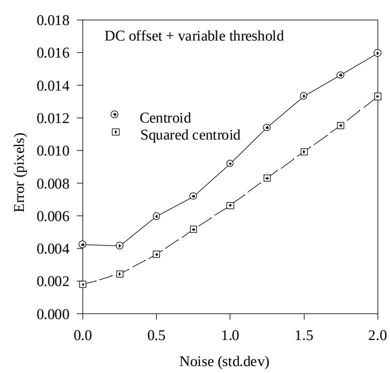


Figure 9(a) Graph of the variation in target location error with noise (no threshold).

Figure 9(b) Graph of the variation in target location error with noise (variable threshold).

Figure 9(c) Graph of the variation in target location error with noise (DC offset + variable threshold).

3.4 Some observations.

The squared centroid method appears to outperform the centroid method in most tests that it is applied to. Care must be taken when using this algorithm to understand its limitations when large, flat top targets are used. The use of a DC offset in the simulations appears to provide some results that are markedly different from those without it. It is thought that this situation is a good model which most closely fits the practical situation found in most CCD camera / frame-grabber combinations. This effect requires further investigation as it is not always clear the level which has been set in a given frame-grabber. If a frame-grabber has an adjustment for this DC offset it is recommended that the signal noise, with no light input to the camera, is brought up sufficiently to just stop producing noise in the image. Care must be taken not to confuse this effect with timing errors such as line-jitter as the location errors are just as clearly produced in the **y** direction as in the **x** direction and are systematic. Further investigations are warranted into the errors produced when inappropriate thresholds are used. These errors are particularly inconvenient as they are also systematic and related to the non-symmetry of target images. In the tests on noise a variable level of noise was added to all intensity levels. It is recognised that this is not a correct model of what happens in practise as the noise level increases with intensity level. This model was incorporated into the simulation for size and threshold but unfortunately it was not possible to incorporate the results for the noise tests in this paper.

4. THE USE OF THE FOURIER TRANSFORM IN THE SIMULATION.

The Fourier Transform method of combining targets and backgrounds gives a useful technique for analysing various effects that are difficult to test in practise. The model of the OTF and the target and background images were multiplied in the frequency domain and the inverse Fourier Transform performed. This method was used to test the effect of various background types on target location accuracy. For all of the tests the size of the input target cylinder diameter was five pixels and the OTF had an equivalent standard deviation of a Gaussian function of 1.2.

4.1 Sloping intensity background and step change intensity variation.

A number of sloping backgrounds were constructed and the cylindrical target discussed in the previous section was added to them. The Fourier Transform process was performed for each of these combinations and the location of the target using the centroid method was compared with its known true location for a variety of threshold levels.

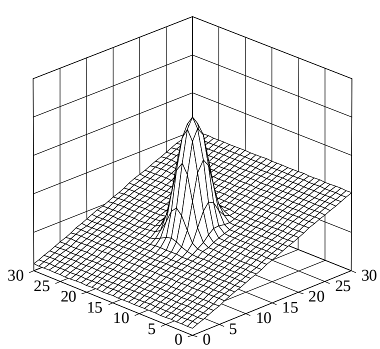


Figure 10(a) A sloping intensity background with convolved target.

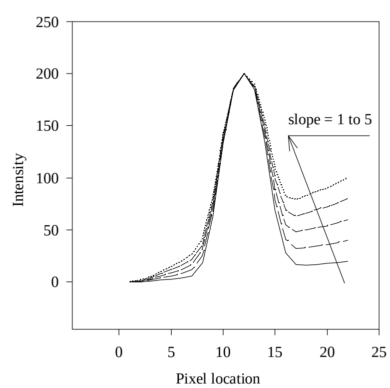


Figure 10(b) A number of cross-sections of the convolved background and targets.

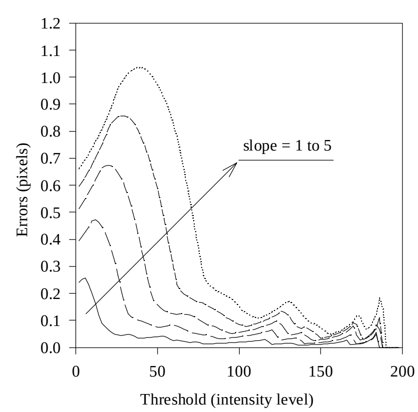


Figure 10(c) Graph of errors in target location for various threshold values.

For each of the sloping backgrounds the location of the target was computed using the centroid method. A window of size 11x11 pixels was used. The error of location was plotted against various thresholds for these images. When the threshold was lower than the background level the edge of the window would have included the background intensity values. Hence, the left hand side of the figure shows large errors in target location. The point at which the threshold was bigger than the background can be clearly seen in the figure where for the case of the gradient of 1 this point occurs at a threshold of approximately 25. Two further observations are possible from this graph: (a) as the threshold is increased so the error in location decreases until the effect of the decrease in the number of intensity observations becomes more significant, (b) the greater the slope of the background the greater the error in location for all threshold values. It can be concluded that it is not advisable to place a target in a position where the background intensity is uneven as an error in location will result. However, if it is unavoidable then a higher than normal threshold could be used provided that the resulting target image is not so small that it causes increased errors in target location. A number of tests were also conducted to analyse the effect of a step change in background intensity. This probably represents the worst case error that it is possible to have in practise. The same methodology was used and the results are illustrated in figure 11 (a) - (c).

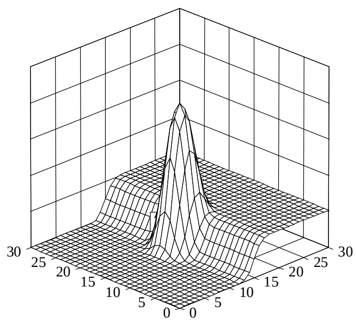


Figure 11(a) A target convolved with a step background.

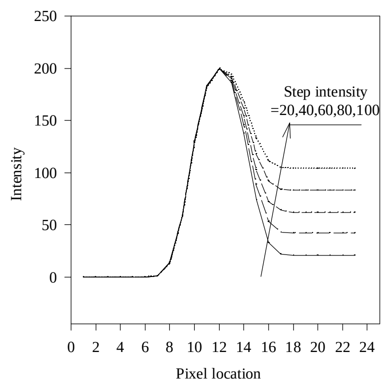


Figure 11(b) A number of cross sections of the simulated target images.

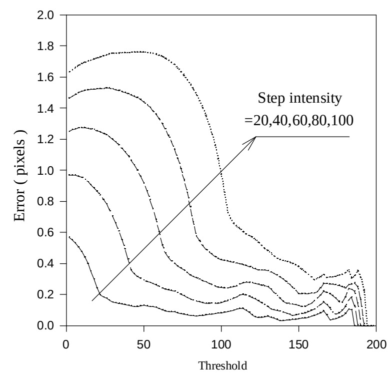
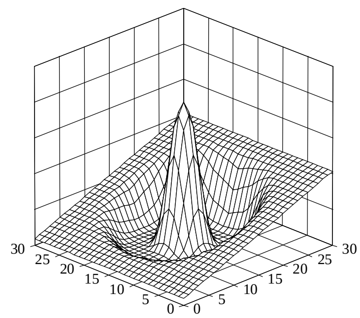


Figure 11(c) A graph of the errors in location of the simulated target images.

Figure 11(a) illustrates a simulation test image with a step changes in background illumination. Five levels were used to investigate the effect, the cross-sections of which are illustrated in figure 11(b). The results of computing the centroid of the target are given in figure 11(c). For the case of a step change in background intensity the error in location is even more pronounced than for a gradual change in intensity levels but the trends and observations are similar.

4.2. Investigation of various size black target backgrounds.

A simulation was performed to analyse the effect of a black target background placed around the target. The same methods were used to analyse the effect of changing the black target background size. Figure 12(a) illustrates one of the simulation cases where a sloping intensity background with gradient 3, a black background, and a target are added together and convolved with the optical transfer function.



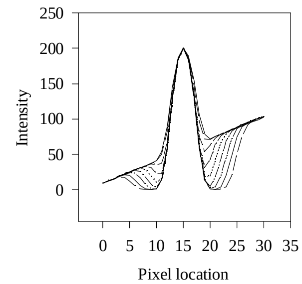
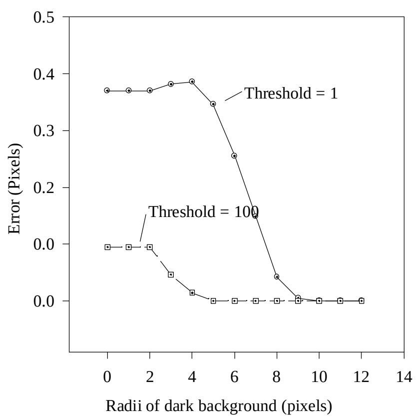


Figure 12(a) The convolution of a target, and a sloping intensity background with the OTF of a lens.

Figure 12(b) The effect of varying the size of the background.

Figure 12(b) illustrates that as the background gets larger then the influence of the sloping intensity background is reduced.



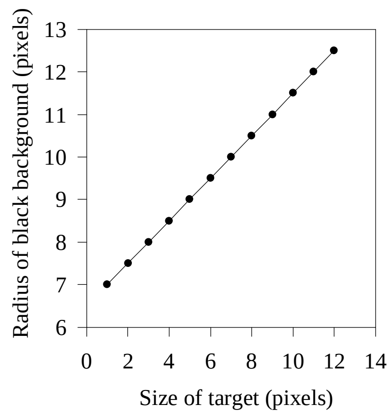


Figure 13(a) The variation in error of location with radii of background of the target.

Figure 13(b) The variation is diameter of the background with target diameter.

This effect is illustrated in figure 13(a) where the target location of the centroid of the target has been computed at two different threshold levels. A high threshold, compared to the peak intensity mean, ensures that the effect of the background is minimised but a higher error in location should be expected due to the reduced size of the thresholded target. For the case of a low threshold, a minimum radius of the background is required for the sloping intensity background to have minimum effect. For the standard deviation used in the simulation this occurs at a radius of nine pixels which was found to be consistent in a large number of tests that were conducted. By referring to the radius of the background in image space means that the physical target size is independent of the distance of the target from the camera. A larger background will be required the further the target is away from the camera. A graph of the radius of the background for a number of differing target sizes is illustrated in figure 13(b). The difference between the radius of the target and the radius of the background at a point where there is no interference between the sloping intensity background and the target location is a constant which is related to the particular lens/camera combination that is used. The same arguments can be used to determine the closest that two target should be to each other in a given image to avoid mutual interference as shown in figure 14(a). For the situation modelled in this paper a graph of the distance between targets is given in figure 14(b).

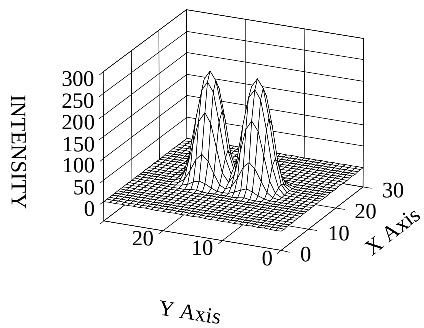


Figure 14(a) Two targets close together.

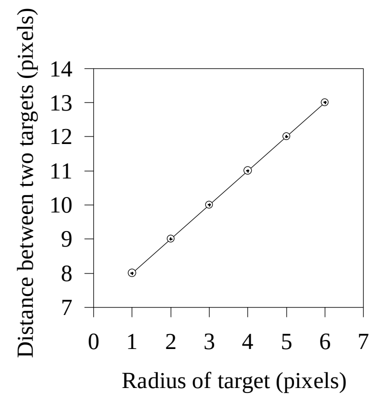


Figure 14(b) The minimum distance between targets with increase of target radius.

5. PRACTICAL TESTS.

The results from both of the simulations were tested in practise by producing the conditions that were modelled in the simulations. Only a few examples were chosen due to the difficulty in constructing such practical models.

5.1 Comparison between simulation and practical tests.

To test the measurement precision of the centroid and squared centroid methods a target was placed onto a background that could be moved by a stepper motor driven translation stage. Measurements of the location of the targets were taken at regular time intervals until the target had moved by approximately nine pixels. The linearity of both variables was verified by the straightness of the line which was analysed by plotting the residuals from a least squares best fit straight line applied to the data. Figure 15(a) illustrates the results from the centroid methods and figure 15 (b) illustrates the results from the squared centroid method.

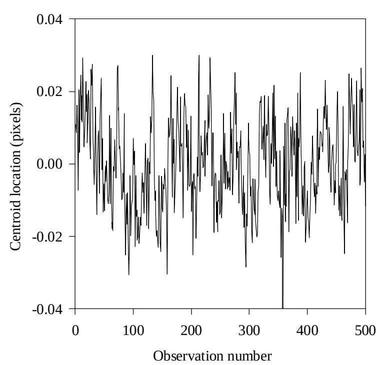


Figure 15(a) Errors in centroid location for a moving target.

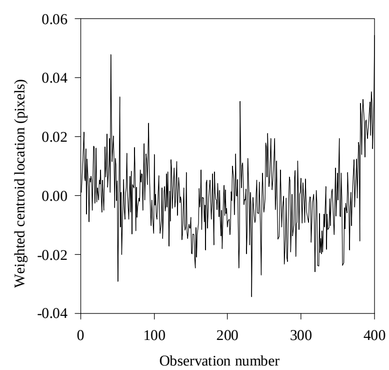


Figure 15(b) Errors in squared centroid location for a moving target.

Figure 16 (a) & (b) are histograms of the frequency distribution of figures 15(a) & 15(b) respectively.

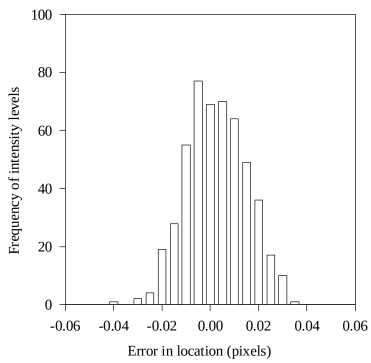


Figure 16(a) Histogram of location errors for centroid method.

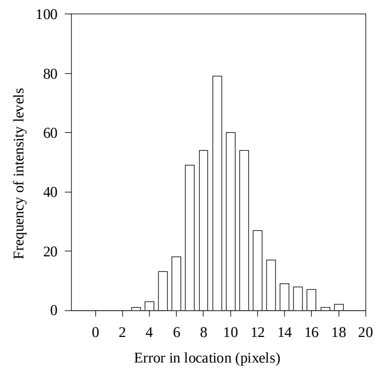


Figure 16(b) Histogram of errors for squared centroid method.

The standard deviation of the measurements was 0.0126 and 0.0125 pixels for the centroid and weighted centroid method respectively. The simulation results for these cases were 0.0062 and 0.005 respectively. The histograms show that for this relatively small sample the distribution of errors has an approximately normal distribution. Furthermore, there is little difference between the two methods in this case. However, in both cases these data were collected with the target moving in the ‘x’ axis so these results include line-jitter and other timing related effect. By comparison with the simulation results for this and other sets where the ‘y’ axis was also taken into account, results within approximately 30% were common. The difference between the ‘x’ and ‘y’ axis results attributable to timing effects in the EPIX frame-grabber was approximately 0.01 of a pixel.

5.2. A comparison of the Fourier Transform method and practical tests.

A background was constructed of a black and white strip, the illumination was arranged so that the background for the black section was around zero grey levels and the white section had an intensity of approximately 80 grey levels. A target was placed on a glass plate such that the peak intensity was approximately 230 grey levels. The black/white edge was moved from one side of the target to the other and the centroid measured under three conditions of no target background, a 2mm. target background, and a 11mm. target background. The results for these tests are illustrated in figures 17(a) - (c).

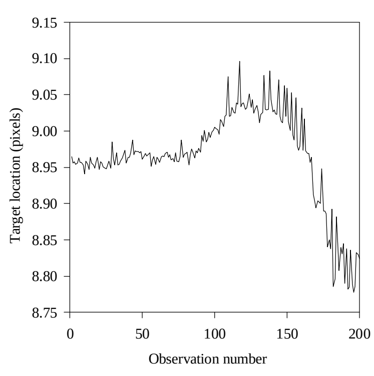


Figure 17(a) Target centroid location (no background).

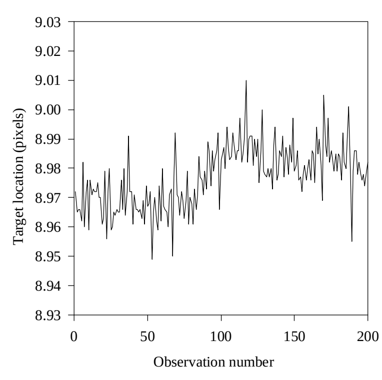


Figure 17(b) Target centroid location (2mm. background).

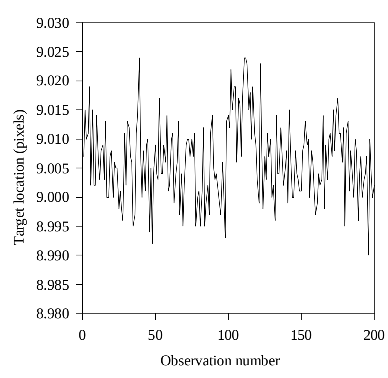


Figure 17(c) Target centroid location (11mm background).

The results illustrate what was simulated using the Fourier Transform methods. A step change in background of a target has a definite influence on the reported location of the target (>0.2 pixels). A relatively small background is able to reduce the effect to < 0.02 pixels. An appropriate size background can nullify the background effect entirely. It should be noted that these tests were carried out using the squared centroid method whereas the simulation used the centroid method. Hence, the errors in this practical case could be expected to be less than the simulation case. By comparison with figure 11(c) the results of the real situation are broadly comparable with those of the simulation, and if the centroid method had been used in the practical tests then the results would be expected to be closer together.

6. CONCLUSIONS.

The research conducted for this paper has analysed some of the potential sources of error that occur when targeted objects are observed from various viewpoints. These errors can occur when a target is placed on a background that is varying in intensity or has a step change in intensity. While in many cases targets will be used that will have a black background surrounding the targets, because of the large size of such targets it is not desirable to use them if the object is changing in shape rapidly. The research concludes that such backgrounds certainly influence the location of a target when measured using the centroid method. An appropriate background size can be used to completely nullify the background effect, or if a small target is required, a higher than normal threshold may also reduce the location error, but at the expense of additional errors due to the loss of intensity information from the target. In addition, results from simulation experiments are also given which show that modelling the measured noise from the frame-grabber/camera combination in the simulation produces results which are very close to the errors experienced in practise. This type of work is useful in providing a means of scrutinising where algorithms produce unwanted errors and in predicting the size of a target background or the threshold that should be used in given circumstances. However, it must be noted that when such standard deviations are applied to target observations in bundle adjustments these errors are very small compared to the those which tend to be used in practise. This is likely to be because there are other unmodelled effects in the small format CCD camera/lens combinations which are much larger than these relatively small errors. Other ongoing work is investigating these errors to attempt to ensure that the level of precision that it is possible to obtain for each camera is able to be used in practise.

7. REFERENCES.

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